PART I

1. A short history of ice:
a mechanicists point of view

2. From the discrete to the continuous:
   homogenization & the continuum hypothesis

3. πάντα ρειν – everything flows: Rheology 1
1. A short history of ice: a mechanicists point of view

Ice plays a prominent role in the evolution of material science and crystallography

- according to ancient belief, stone ice is ice frozen in a permanently rocky state

- stone ice (SiO₂) and H₂O ice are referred to as κρισταλλόσ (crystals, translating to 'ice')

- Cardano launches of the science of ice (1550): stone ice and H₂O ice are distinct as they behave differently under fire
1. A short history of ice: a mechanicists point of view

the resolution of the hexagonal structure of ice involved ...

Kepler (1611), explaining the hexagonal structure of ice in terms of cubic and hexagonal packings of spheres
1. A short history of ice: a mechanist's point of view

the resolution of the hexagonal structure of ice involved ...

Hooke (1665, "Micrographia"), observing the hexagonal structure of ice under a microscope
1. A short history of ice: a mechanicists point of view

the resolution of the hexagonal structure of ice involved ...

Dalton (1808), expressing the symmetry of ice crystals as a consequence of the atomistic structure of matter (supporters: Boyle, Newton, Lomonosov,...)
1. A short history of ice: a mechanicists point of view

the resolution of the hexagonal structure of ice involved ...

19th century mathematicians Bravais, Cauchy & Schönflies,...

developing the group theoretical foundations of crystallography

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Bravais‘
14 lattices

Cauchy

Bravais

Schönflies

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1. A short history of ice: a mechanicists point of view

the resolution of the hexagonal structure of ice involved ...

Bragg & Bragg (1922), Pauling (1935), presenting the atomistic structure of hexagonal ice

(hypothetical, perfect crystal)

view along $c$–axis

transversal view
2. From the discrete to the continuous:
   homogenization and the continuum hypothesis

How do we get from the discrete to the continuous
2. From the discrete to the continuous:
   homogenization and the continuum hypothesis

? Is a distant point of view sufficient to go from the discrete to the continuous

G.-P.-Seurat, Circus Sideshow, 1887/88, oil on canvas, Metropolitan Museum of Art, www.metmuseum.org
2. From the discrete to the continuous: homogenization and the continuum hypothesis

Mathematics plays a prominent role in the evolution of continuum mechanics:

Mathematics in the 17th century:

• Leibniz (1660) & Newton (1666/1693): "Calculus"

• rapidly developing disciplines: theories of infinite series, ordinary and partial differential equations, calculus of variations, differential geometry

• applications: cartography, navigation, ballistics, marine & mechanical engineering, mechanics, astronomy
2. From the discrete to the continuous: homogenization and the continuum hypothesis

Mathematics plays a prominent role in the evolution of continuum mechanics:

Mathematics in the 18th century:

• Bernoulli, *Hydraulica*, 1743

• Euler, *Introductio in analysin infinitorum* 1748, *Institutiones calculi differentialis*, 1755

• Lagrange, *Mecanique analytique*, 1788
2. From the discrete to the continuous: homogenization and the continuum hypothesis

Mathematics plays a prominent role in the evolution of continuum mechanics:

Mathematics in the 19th century: Cauchy

- *Cours d'analyse*, 1821
- *Le calcul infinitesimal*, 1823

- Invention of the stress tensor

- Combination of the stress tensor with Euler's laws of mechanics

  general framework for the description of the motion of any continuous medium
2. From the discrete to the continuous: homogenization and the continuum hypothesis

"We shall suppose, [...], that the macroscopic behavior of fluids is the same as if they were perfectly continuous in structure; and physical quantities such as the mass and momentum associated with the matter contained within a given small volume will be regarded as spread uniformly over that volume instead of, as in strict reality, being concentrated in a small fraction of it."

The continuum hypothesis as formulated by Batchelor, Introduction to Fluid Mechanics, 1967
2. From the discrete to the continuous: homogenization and the continuum hypothesis
2. From the discrete to the continuous: homogenization and the continuum hypothesis

“Smallest given volume” in large natural ice masses: polycrystals

(unresolved) randomly oriented hexagonal ice crystals implying mechanically isotropic material behavior
2. From the discrete to the continuous: homogenization and the continuum hypothesis

Postulate: Large natural ice masses are treated as a

- single-constituent (no impurities [dust, minerals], no water...),
- homogeneous (no bubbles,...),
- polycrystalline,
- isotropic

continuous material.
3. πάντα ρείν – everything flows: Rheology 1

Postulate:
the flow behavior of a large natural ice mass is modeled by regarding it as an incompressible, viscous, heat-conducting non-Newtonian fluid

Is rheology a matter of scale?
3. πάντα ρείν  – everything flows: Rheology 1

The rheology (the flow law) has been formulated for the idealized continuum

"single-constituent, homogeneous"
3. πάντα ρείν – everything flows: Rheology 1

The rheology (the flow law) has been formulated for the idealized continuum

"single-constituent, homogeneous"

3. παντα ρειν – everything flows: Rheology 1

The rheology (the flow law) has been formulated for the idealized continuum "isotropic"
3. πάντα ρειν – everything flows: Rheology 1

The rheology (the flow law) has been formulated for the idealized continuum from: Thorsteinsson et al., Ber. Polarforschung, 1996

"isotropic"
3. πάντα ρείν – everything flows: Rheology 1

The rheology (the flow law) has been formulated for the idealized continuum "isotropic"

we observe *anisotropy*:

\[ \alpha \neq \text{not} \]
\[ \sigma \sigma \sigma = \text{identical} \]
\[ \tau \varepsilon \varepsilon \varepsilon \text{v} = \text{turn around} \]
3. πάντα ρειν – everything flows: Rheology 1

Isotropy vs. Anisotropy: Why should we bother?

from: Petit et. al, Nature (399), 1999
3. Πάντα ρείν – everything flows: Rheology 1

Evolving anisotropy (distribution of c–axis orientation) changes the mechanical response of ice and results in *altered*

- flow velocities
- particle positions
- depth–age relation for ice cores
- reconstructions of past climates

*numerical determination of ice age in cores based on models with/without induced anisotropy may differ by 40–100 kyr*

*From: Mangeney et al., 1997*
3. τα παντα ρειν – everything flows: Rheology 1

- deformation of ice sheets takes place
  - under the action of gravity
  - in the presence of high temperatures
  - in response to processes at their boundaries
  - ...

Rheology ...

- depends on Flow–Structure–Environment–Interplay (FSEI)
  [the entire is more than the sum of its parts]
- requires the consideration of *induced* anisotropy
3. παντα ρειν – everything flows: Rheology 1

Rheology is a matter of scale

isotropic flow law
3. παντα ρειν – everything flows: Rheology 1

Isotropic flow law: Part II

Anisotropic flow law: Part III

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3. παντα ρειν – everything flows: Rheology 1

A warning before the break: a true anisotropic flow law is not obtained by introducing enhancement factors to an isotropic flow law

Exercise: calculate the stresses!