Why gradients (and therefore adjoints) are good for you...

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*Estimating the Circulation and Climate of the Ocean (ECCO)*

http://ecco-group.org
Some background

What I do, and how it ties to ice sheet modeling...
Some context: Climate modeling and prediction: What are we trying to do, and what do we know?

- How well do we know the past and present state of the ocean?
- How well do we need to know it in order to make “reliable” (or better) predictions?
The ocean - a laminar vs. an inherently turbulent fluid

Bower et al., Nature, 2009: Deep Western Boundary Current vs. interior pathways, calls into question the conveyor as a useful concept.

The “great ocean conveyor belt” as conceptualized by W. Broecker in the 1980s. The underlying notion seems to be that of the ocean as a slowly moving laminar flow.
Two knowledge reservoirs:

1. Observations
Satellite altimetry, gravity, and regional/global sea-level (1)

Geoid = Earth’s gravity equipotential

Dynamic ocean topography = Surface height changes due to...
Sea level anomalies from Dynamic Ocean Topography reflect changes in:
- ocean currents
- mesoscale eddies and waves
- heating/cooling (forcing)
- freshwater input (precipitation, ice melt)

→ SLA changes due to
- thermo-/halo-steric changes
- mass changes

global-mean sea level trends from satellite altimetry 1992 to 2007 (3 mm/yr)
The Argo float profiling program
(fully deployed since 2004)

**BUT:** doesn’t measure…
- below ~2000 m depth
- near boundaries and marginal seas
- under sea-ice
More satellite sea surface measurements:
SST, sea-ice concentration, wind speed, ocean color and chlorophyll

SST from passive microwave radiometers such as SMMR, SSM/I, AMSR-E

SeaWIFs: Sea-viewing Wide Field-of-view sensor measuring ocean color should provide information on ocean phytoplankton and ocean net primary production (i.e. biological activity)

Daily sea ice concentration fields from passive microwave radiometers (since 1979)

Surface wind speeds from QuikSCAT radar scatterometer
Two knowledge reservoirs

2. Theory:
Modeling the time-evolving three-dimensional ocean with a general circulation model (GCM)

An approximated form of the Navier-Stokes equations for an incompressible fluid, consisting of:

- **momentum equation (including Coriolis term)**
- **conservation of mass (continuity equation)**
- **conservation of tracers (heat, salt)**
- **equation of state**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\frac{D\vec{v}_h}{Dt} + f\hat{k} \times \vec{v}_h + \frac{1}{\rho_c} \nabla z p = \vec{F}$</td>
<td></td>
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<tr>
<td>$\epsilon_{nh} \frac{Dw}{Dt} + \frac{g \rho}{\rho_c} + \frac{1}{\rho_c} \frac{\partial p}{\partial z} = \epsilon_{nh} F_w$</td>
<td></td>
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<tr>
<td>$\nabla_z \cdot \vec{v}_h + \frac{\partial w}{\partial y} = 0$</td>
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<tr>
<td>$\rho = \rho(\theta, S)$</td>
<td></td>
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<tr>
<td>$\frac{D\theta}{Dt} = Q_{\theta}$</td>
<td></td>
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<tr>
<td>$\frac{DS}{Dt} = Q_s$</td>
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</tbody>
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Physical state of the ocean comprising:

- **temperature**
- **salinity**
- **horizontal and vertical velocity**
- **sea surface elevation**

are discretized on a horizontal/vertical mesh, and stepped forward in time.

http://mitgcm.org
State estimation: Formally combining the two knowledge reservoirs
An early vision, ca. 1982:

C. Garrett and C. Wunsch, Eds., SIO Reference Series 84-5, March 1984
The optimal control approach: Iterative optimization via gradient obtained from *adjoint model*.
Least-squares optimization / optimal control problem

Given:
- a set of (possibly different types of) observations
- a numerical model & set of initial/boundary conditions, parameters

Question: (estimation / optimal control problem)
Find “optimal” model trajectory consistent with available observations

Approach: seek minimum of least square cost function

$$\min_{\vec{u}} \{ J(\vec{u}) \} = \min_{\vec{u}} \left\{ \sum_i \left[ \text{model}_i(\vec{u}) - \text{data}_i \right]^2 \right\}$$

$$\rightarrow$$ seek $\vec{\nabla}_u J(\vec{u})$ to infer update $\Delta \vec{u}$ from variation of controls $\vec{u}$

$$\vec{u}^{n+1} = \vec{u}^n + \Delta \vec{u}$$

Results: see ECCO (Stammer et al., 2002/03)
- optimal/consistent ocean state estimate
- adjusted initial/boundary value estimates
Adjoint-based sensitivity calculations
(e.g. ice sheet volume sensitivities)

- Finite difference approach:
  - Take “guessed” anomaly (e.g. **basal sliding** $b_s$) and determine its impact on model output (ice volume $V$)
  - Perturb each input element $b_s(i, j)$ to determine its impact on output $V$.

  Impact of **one input to all outputs**

- Reverse/adjoint approach:
  - Calculates “full” sensitivity field
    \[
    \frac{\partial \text{ice volume}}{\partial \text{basal sliding} (x, y, t)}
    \]
  - Approach: Let
    \[
    \mathcal{J} = V, \quad \bar{u} = b_s(i, j)
    \]
    \[
    \vec{\nabla}_u \mathcal{J} (\bar{u}) = \frac{\partial V}{\partial b_s(x, y, t)}
    \]

  Sensitivity of **one output to all inputs**
The idea to do something with an adjoint ice sheet model...

Host at the Bjerknes Centre (Norway) where work got started

Co-author

Fennoscandian ice sheet (Folgefonna)
Model parameter sensitivities: basal conditions
Heimbach & Bugnion, Annals Glaciol., 2009

**Basal melt rate sensitivity**
- Reference basal temperature [°C]
- Reference basal melt rate [kg/m²/s]

**Basal sliding sensitivity**
- Reference $C_{sl}$ 11.2 m/a/Pa
The remainder of the lecture...

- What is an adjoint.
- Automatic differentiation: one way to get the adjoint.
- Some examples.